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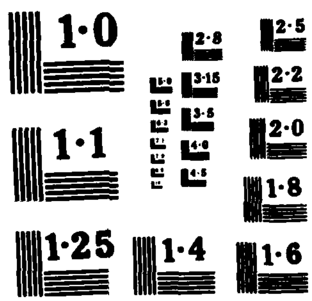
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ACCELERATION-INVARIANT APPROXIMATION METHOD FOR RECURSIVE DIGITAL FILTERS

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by

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ABSTRACT

This document describes a procedure for designing recursive digital filters from continuous-time filters when the ratio of the sampling frequency to the pole frequency is small. The discrete-time transfer function of digital filters obtained by this method, called the acceleration-invariant transformation, has been derived from the partial fraction expansion of the continuous-time transfer function. An error function between the digital and analog transfer functions has also been deduced. Finally, the performance of the proposed method is demonstrated by plotting the frequency response of high-order Butterworth and elliptic filters.

RÉSUMÉ

Ce document décrit une méthode pour concevoir des filtres numériques récursifs à partir des filtres analogiques lorsque le rapport entre le taux d'échantillonnage et la fréquence du pôle est faible. La fonction de transfert en z des filtres numériques obtenus par cette méthode, appelée l'invariance à l'accélération, a été déduite de la décomposition en fraction partielle de la fonction de transfert des filtres analogiques. Une fonction qui permet de calculer l'erreur entre la fonction de transfert numérique et la fonction de transfert analogique a aussi été déduite. Finalement, la performance de cette méthode est démontrée en traçant la réponse en fréquence des filtres Butterworth et elliptiques à ordres élevés.

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1.0 INTRODUCTION

This document describes an alternative approximation method for recursive digital filters when the ratio of the sampling frequency to the cutoff or center frequency (f_s/f_c) is small. Recursive digital filters are usually designed by using the impulse-invariant method and the bilinear transformation. These methods, which constitute textbook material (Refs. 1 to 5), permit the derivation of a digital filter from a suitable analog filter satisfying a set of prescribed specifications. They also provide valid matches with their corresponding analog filters when the ratio f_s/f_c is sufficiently high. At low f_s/f_c, these methods have serious drawbacks that lead to inappropriate approximations.

The standard-z transformation (also called the impulse-invariant transformation) is a method whereby the response of the derived digital filter to an impulse is identical to the sampled impulse response of the continuous-time filter. This method yields valid matches with the corresponding analog filter only at high sampling rates and it is satisfactory when the analog system is sufficiently band-limited prior to transformation. At low ratios of f_s/f_c, this method can be applied only to transfer functions in which the denominator degree exceeds that of the numerator by at least two. The aliasing effects obtained by this transformation render it entirely useless for digital high-pass and band-reject filters. In addition, for elliptic low-pass and band-pass filters, the equiripple character of the stop band is destroyed by the aliasing effect. The bilinearly transformed filters are essentially identical to the original analog filter when the ratio f_s/f_c is greater than 10. However, this transformation results in a nonlinear relationship between the analog and the digital frequencies. It implies that the frequency response of the continuous system being transformed will be a warped version of the analog frequency response. This effect, which is observed particularly when the ratio f_s/f_c is smaller than 10, renders the method quite complicated when designing adaptive filters. In addition, neither the impulse response nor the

phase response of the analog filter is preserved in a digital filter obtained by the bilinear transformation.

Other methods of deriving recursive forms of continuous-time transfer functions of analog filters are an extension of the concept of the impulse invariance. In the sinusoid invariance (Ref. 6), the step invariance (Refs. 7 and 8) and the ramp invariance (Ref. 8), the digital filter design is optimized for a sequence of sinusoids, steps or ramps joining the sampled values as an input signal. These methods attenuate the aliasing effect encountered in the impulse invariance. The sinusoid-invariant method preserves the phase characteristics of analog filters and it is suitable to the design of filters with a constant group delay. The step-invariant method can be applied to transfer functions in which the degree of the denominator can exceed that of the numerator by at least one. In the ramp-invariant method, the numerator degree can be as high as the denominator degree. In Ref. 8, it is shown that the step and ramp invariances of low-pass and band-pass filters, if they are realized in a parallel form, produce digital filters whose frequency response does not depend on the sampling frequency in the frequency band of 0 to $f_s/2$. The frequency response of the step-invariant high-pass and band-reject filters realized in a parallel or a cascade form is unacceptable for small ratios f_s/f_c because of the aliasing effects. The ramp invariance realized in a cascade form can be used to digitize high-pass and band-reject filters. The ratio f_s/f_c can be decreased below 10 for Butterworth filters but the equiripple character in the stop-band response of elliptic filters is preserved by a ratio of at least 10.

The approximation method described in this report is derived from analog filters such that an acceleration invariant response is maintained. The discrete-time transfer function of the digital filters is obtained from the partial fraction expansion of the continuous-time transfer function to lead directly to a realization in a parallel form. An error function between the transfer functions of the digital and

analog filters has also been deduced. This method is shown to approximate correctly, on a wide bandwidth, the frequency responses of low-pass, band-pass, high-pass and band-reject analog filters for small ratios of f_s/f_c .

This work was performed at DREV between November 1982 and April 1983 under PCN 21J05, Guidance and Control Concepts.

2.0 DERIVATION OF THE ACCELERATION-INVARIANT METHOD

Assume that the coefficients of an analog filter with Laplace transformation (i.e. transfer function) given by

$$H_A(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{i=0}^{M'} b_i s^i}{\sum_{i=0}^{N'} a_i s^i} \qquad [1]$$

are known. Y(s) and X(s) are respectively the Laplace transformation of the output y(t) and the input x(t). M' must be smaller or equal to N'. If we assume that all real and complex poles are simple, eq. 1 can be rewritten in its partial fraction expansion as

$$H(s) = A_o + \sum_{i=1}^{N} \frac{A_i}{s + p_i} \qquad [2]$$

Equation 2 covers low-pass, high-pass, band-pass and band-reject filters among the well-known classes such as the Butterworth, Bessel, Chebyshev and elliptic filters.

The s domain filter response of eq. 2 to an acceleration input is written as

$$Y_A(s) = H_A(s)X(s) = H_A(s)/s^3 \qquad [3]$$

Substituting eq. 2 into eq. 3 and finding the inverse Laplace transformation of this relation yield the following time function:

$$Y_A(t) = \frac{A_o t^2}{2} + \sum_{i=1}^{N} \left\{ \frac{A_i t^2}{2p_i} - \frac{A_i t}{P_i^2} + \frac{A_i(1-e^{-P_i t})}{P_i^3} \right\} \qquad [4]$$

Hence, the sampled sequence of $Y_A(t)$ can be obtained by applying the z transformation to eq. 4. This gives:

$$Y_D(z) = \frac{A_o T^2 z^{-1}(1 + z^{-1})}{2(1 - z^{-1})^3} + \sum_{i=1}^{N} \left\{ \frac{A_i T^2 z^{-1}(1 + z^{-1})}{2p_i(1 - z^{-1})^3} \right.$$

$$\left. - \frac{A_i T z^{-1}}{P_i^2(1 - z^{-1})^3} + \frac{A_i z^{-1}(1 - e^{-P_i T})}{p_i^3(1 - z^{-1})(1 - z^{-1}e^{-P_i T})} \right\} \qquad [5]$$

where $z= e^{Ts}$ and is the sampling period. $Y_D(z)$ can also be expressed as

$$Y_D(z) = H_D(z) \cdot X(z) \qquad [6]$$

where $H_D(z)$ is the discrete-time transfer function of the digital filter. $X(z)$, the sampled sequence of the acceleration input, is

$$X(z) = \frac{T^2 z^{-1}(1 + z^{-1})}{2(1 - z^{-1})^3} \qquad [7]$$

Hence, $H_D(z)$ is derived by combining eqs. 5, 6 and 7. This operation yields

$$H_D(z) = \frac{Y(z)}{X(z)} = A_o + \sum_{i=1}^{N} \frac{C_i z^{-2} + D_i z^{-1} + E_i}{(1 + z^{-1})(1 - e^{-P_i T} z^{-1})} \qquad [8]$$

where $C_i = -\left(\frac{A_i}{P_i} + \frac{2A_i}{P_i^2 T}\right) e^{-P_i T} + \frac{2A_i}{P_i^3 T^2}\left(1 + e^{-P_i T}\right)$

$$D_i = \left(\frac{A_i}{P_i} - \frac{4A_i}{P_i^3 T^2}\right)\left(1 - e^{-P_i T}\right) + \frac{2A_i\left(1 + e^{-P_i T}\right)}{P_i^2 T}$$

$$E_i = \frac{A_i}{P_i} - \frac{2A_i}{P_i^2 T} + \frac{2A_i}{P_i^3 T^2}\left(1 - e^{-P_i T}\right)$$

A digital filter whose response to an acceleration excitation corresponds to that of the analog filter has then been obtained.

3.0 ERROR FUNCTION BETWEEN THE ANALOG AND THE ACCELERATION-INVARIANT DIGITAL FILTER

The purpose of this chapter is to establish an error function between the continuous-time and the discrete-time transfer functions. A relation can be found by regarding the response of the digital filter as the sampled response of the corresponding analog filter. The Fourier transform of y(t) is obtained from Ref. 1 as

$$Y_D(e^{j\omega T}) = \frac{1}{T} \sum_{m=-\infty}^{\infty} Y_A(j\omega + jm\omega_S) \qquad [9]$$

where $\omega_S = 2\pi/T$

The substitution of eq. 9 into eq. 3 gives

$$Y_D(e^{j\omega T}) = \frac{1}{T} \sum_{m=-\infty}^{\infty} \frac{H_A(j\omega + jm\omega_S)}{-j(\omega + m\omega_S)} \qquad [10]$$

The response of the digital filter to an acceleration input excitation, which can be derived from the relation ($Y_D(z) = H_D(z) \cdot X(z)$), gives

$$Y_D(e^{j\omega T}) = \frac{H_D(e^{j\omega T})T^2 e^{-j\omega T}(1+e^{-j\omega T})}{2(1-e^{-j\omega T})^3} \qquad [11]$$

A relation between the analog and digital transfer functions is obtained by substituting eq. 11 into eq. 10. This yields

$$H_D(e^{j\omega T}) = -\frac{2(1 - e^{-j\omega T})^3}{jT^3 e^{-j\omega T}(1 + e^{-j\omega T})} \sum_{m=-\infty}^{\infty} \frac{H_A(j\omega + jm\omega_S)}{(\omega + m\omega_S)^3} \qquad [12]$$

For sufficiently large ω_S, the effect of frequency aliasing can be neglected. Under this condition, eq. 12 can be written as

$$H_D(e^{j\omega T}) = E(e^{j\omega T})H_A(j\omega) \qquad [13]$$

where $E(e^{j\omega T})$ is an error function given by

$$E(e^{j\omega T}) = -\frac{2(1 - e^{-j\omega T})^3}{j\omega^3 T^3 e^{-j\omega T}(1 + e^{-j\omega T})} \qquad [14]$$

The magnitude and phase responses of the error function (eq. 14) have been plotted in Fig. 1 from $f_s/10^4$ to $f_s/2.5$. It is shown that the magnitude response of the digital filter is identical to that of its analog counterpart over the frequency band of $f_s/10^4$ to $f_s/10$. The error in magnitude increases gradually to attain 2.5 dB at $f_s/2.5$. No error in phase is observed from $f_s/10^4$ to $f_s/2.5$. From $f_s/2.5$, the error increases to attain ∞ at $f_s/2$. The acceleration-invariant filters are thus limited to the frequency range of 0 to $f_s/2.5$.

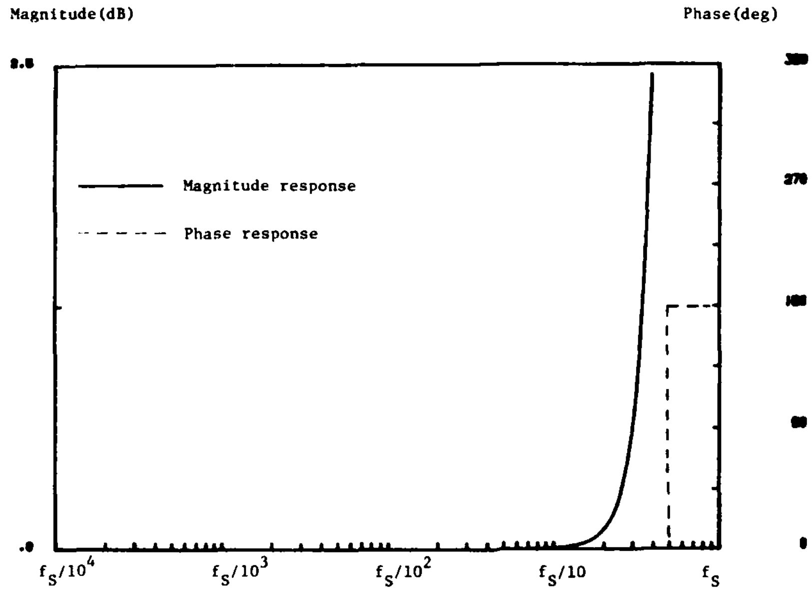

FIGURE 1 - Frequency response of the error function between the analog and digital transfer functions

4.0 PERFORMANCE OF THE ACCELERATION-INVARIANT METHOD FOR HIGH-ORDER BUTTERWORTH AND ELLIPTIC FILTERS

Figures 2 to 5 show the magnitude and phase-frequency responses of high-order Butterworth and elliptic digital filters obtained by the acceleration-invariant method. In all examples, the response of digital filters is presented for various sampling frequencies (100, 10 and 4 kHz) and it is accompanied by the response of the corresponding analog filters. Figures 2 and 3 illustrate fifth-order low-pass and high-pass Butterworth and elliptic filters. The cutoff frequency (f_c) and the gain of these filters are maintained at 1 kHz and 1 respectively. In addition, for elliptic filters, the stop-band attenuation is at least 40 dB and the pass-band ripple is limited to 3 dB. The band-pass and the band-reject Butterworth and elliptic filters of Figs. 4 and 5

were derived from a transformation performed on the corresponding low-pass filters (Ref. 1). The lower and upper cutoff frequencies of these filters are located at 700 Hz and 1.0 kHz. Their center frequency is noted by f_c. The poles and zeros of analog filters are given in Appendix A.

The acceleration invariance of the low-pass and band-pass filters produces digital filters whose frequency response matches very closely that of their analog counterpart in the frequency band of 0 to $f_s/2.5$. In this region, the frequency response does not depend on the sampling frequency. For elliptic filters, the equiripple character of the stop-band response is correctly preserved. The high-pass digital filters are essentially identical to the original analog filter when the ratio f_s/f_c is greater than or equal to 10. When it decreases below 10, the aliasing effect can be observed in the magnitude response of the digital high-pass filters. The Butterworth filter exhibits an error in magnitude and in phase in the low-frequency band. However, when f_s is set to $4f_c$, this error does not exceed 10 dB in magnitude and 20° in phase at $f_c/10$. The high-pass elliptic filter does not perfectly preserve the equiripple character of the stop-band response when f_s/f_c is smaller than 10. The phase response is also slightly disturbed around the resonant frequency. The band-reject digital filters react in the same manner as the high-pass filters. Their frequency responses coincide with those of the corresponding analog filters when the ratio f_s/f_c is greater than or equal to 10. When it becomes smaller than 10, the attenuation of the Butterworth filter, the phase response and the equiripple character of the elliptic filter in the stop-band are not correctly reproduced. Although all the characteristics of the digital high-pass and band-reject filters are not preserved when the ratio f_s/f_c is set to a value smaller than 10, no gain is introduced by the acceleration-invariant transformation. Furthermore, this method does not move the poles of the digital filters as a function of the sampling frequency and it can be applied to a wide frequency band.

Legend for Figures 2 through 5

————————————	Continuous-time filter
------------------	Digital filter when f_s = 100 kHz
•••••••••••••••••••	Digital filter when f_s = 10 kHz
-•-•-•-•-•-•-•-•-•	Digital filter when f_s = 4 kHz

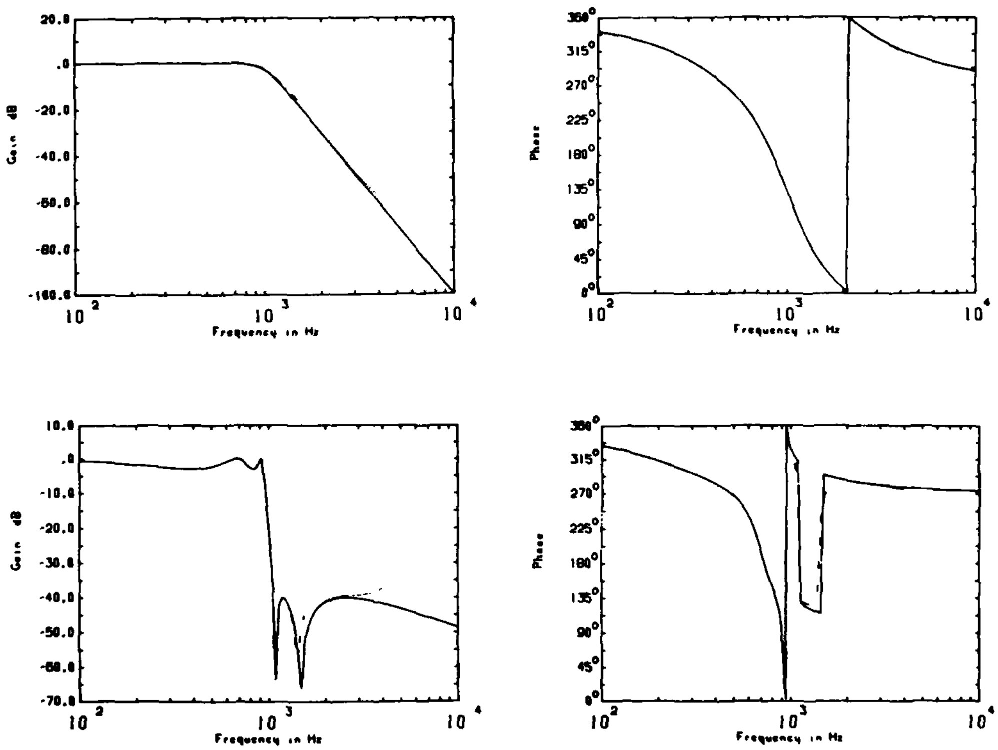

FIGURE 2 - Frequency responses of Butterworth and elliptic low-pass digital filters obtained by the acceleration-invariant method for various ratios f_s/f_c (4, 10, 100)

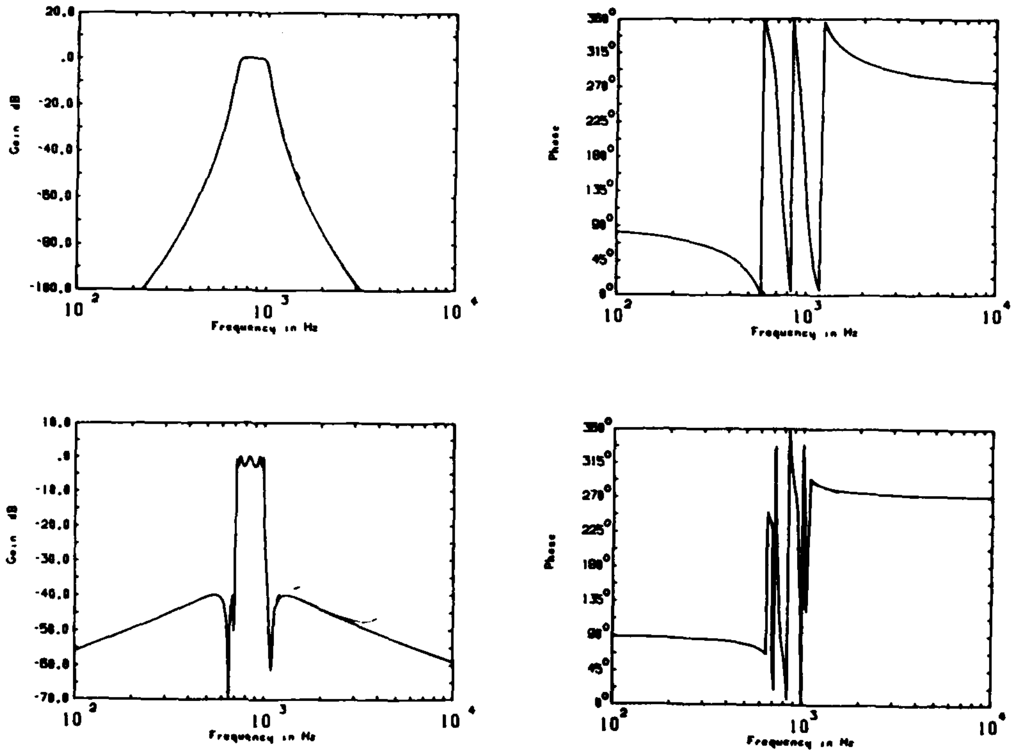

FIGURE 3 - Frequency responses of Butterworth and elliptic band-pass digital filters obtained by the acceleration-invariant method for various ratios f_s/f_c (4, 10, 100)

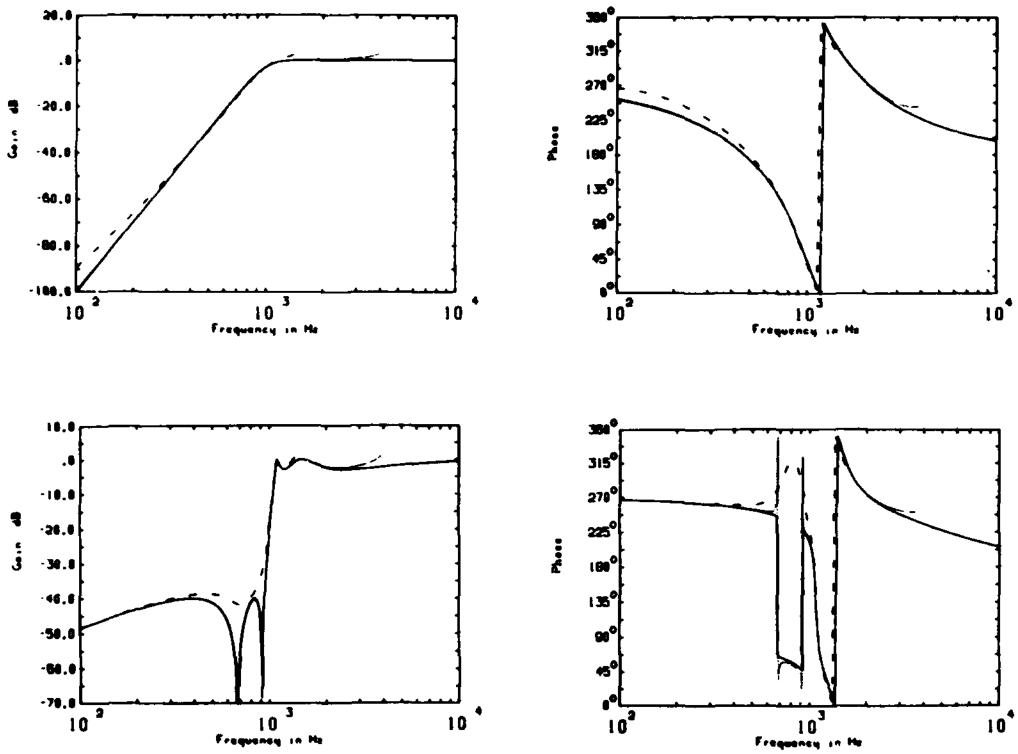

FIGURE 4 - Frequency responses of Butterworth and elliptic high-pass digital filters obtained by the acceleration-invariant method for various ratios f_s/f_c (4, 10, 100)

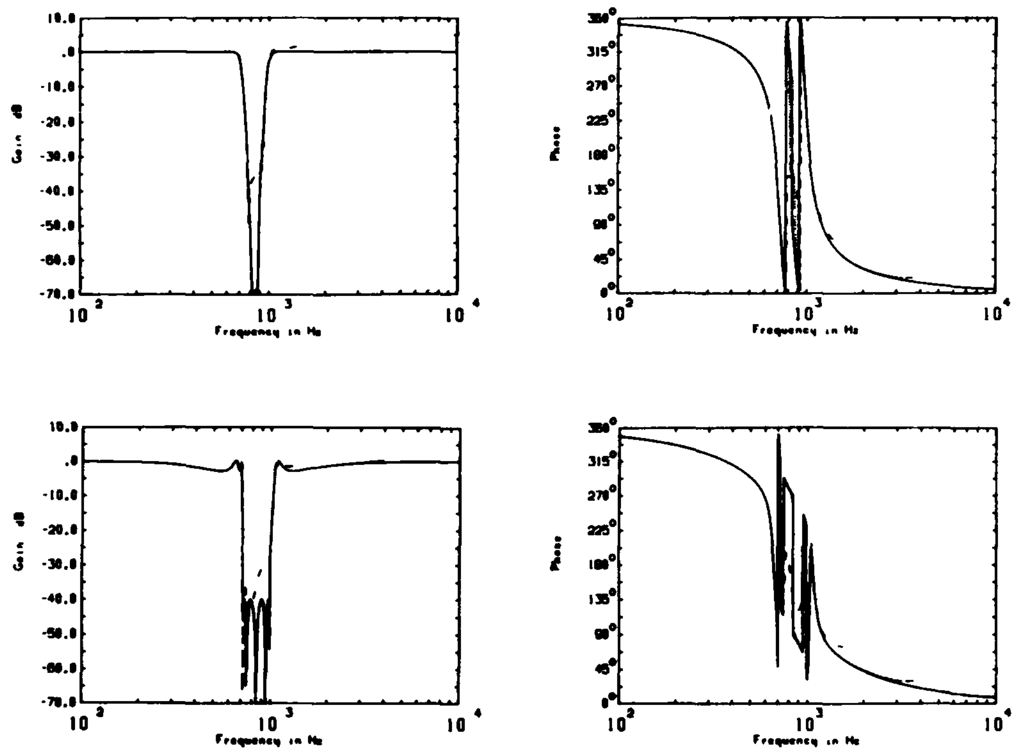

FIGURE 5 - Frequency responses of Butterworth and elliptic band-reject digital filters obtained by the acceleration-invariant method for various ratios f_s/f_c (4, 10, 100)

5.0 CONCLUSION

An approximation method has been described whereby a digital filter can be derived from an analog filter while maintaining an acceleration-invariant response. The discrete-time transfer function of digital filters obtained by this method has been deduced from the partial fraction expansion of the continuous-time filter yielding a parallel realization.

The performance of the proposed method has been determined by giving the frequency response of high-order Butterworth and elliptic filters. The acceleration invariance of low-pass and band-pass filters produces digital filters whose frequency responses match perfectly those of their analog counterparts. Also, these responses do not depend on the ratio f_s/f_c in the frequency band of 0 to $f_s/2.5$. An accurate frequency response of high-pass and band-reject filters can be obtained when the ratio f_s/f_c is maintained equal to or greater than 10. When it decreases below 10, the frequency response of the digital filters remains acceptable but the equiripple character of the elliptic filter in the stop-band, the magnitude response of the Butterworth high-pass filter in the low-frequency band and the peak stop-band attenuation of the Butterworth band-reject filter are not perfectly reproduced by the transformation.

The acceleration-invariant method is then less sensitive to the frequency aliasing in comparison with the impulse-invariant, the step-invariant and the ramp-invariant methods. In addition, this method exhibits no frequency warping, a drawback met in the bilinear transformation. However, this improvement produces an increase in the order of the acceleration-invariant filter. The numerator and denominator degrees of the acceleration-invariant filter exceed by one those of the ramp-invariant or the bilinearly transformed filter.

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APPENDIX A

Continuous-Time Transfer Functions of the Butterworth and Elliptic Filters

This appendix gives the continuous-time transfer functions of the Butterworth and elliptic filters used in Chapter 4.

The transfer functions of these filters are written as

$$H(s) = K_s \frac{\prod_{i=1}^{M'} (s + m_i)}{\prod_{i=1}^{N'} (s + p_i)} \qquad [A.1]$$

1) Fifth-order Butterworth low-pass filter:

- Cutoff frequency = 1000 Hz
  - M' = number of zeros = 0
  - N' = number of poles = 5
  - $K_s = 9.79 \times 10^{18}$
  - $p_{1,2} = 1941 \pm j\ 5975$
  - $p_{3,4} = 5083 \pm j\ 3693$
  - $p_5 = 6283$

2) Butterworth high-pass filter derived from the low-pass filter:

  - Cutoff frequency = 1000 Hz
  - $M' = 5$, $N' = 5$, $K_s = 1$
  - poles are identical to those of the low-pass filter
  - zeros are all located at 0

3) Butterworth band-pass filter derived from the low-pass filter:

- lower cutoff frequency = 700 Hz
- upper cutoff frequency = 1000 Hz
- $M' = 5$, $N' = 10$, $K_s = 2.37 \times 10^{16}$
- zeros are all located at 0
- $p_{1,2} = 340 \pm 6221$
  $p_{3,4} = 242 \pm 4428$
  $p_{5,6} = 843 \pm 5785$
  $p_{7,8} = 682 \pm 4677$
  $p_{9,10} = 942 \pm 5172$

4) Butterworth band-reject filter derived from the low-pass filter:

- lower cutoff frequency = 700 Hz
- upper cutoff frequency = 1000 Hz
- $M' = 10$, $N' = 10$, $K_s = 1$
- poles are identical to those of the band-pass filter
- zeros are located at $0 \pm j\ 5257$

5) Fifth-order elliptic low-pass filter:

- cutoff frequency = 1000 Hz
- $M' = 4$, $N' = 5$, $k_s = 240$
- $m_{1,2} = 0 \pm 9309$, $m_{3,4} = 0 \pm j\ 6865$
- $p_{1,2} = 775.2 \pm j\ 4367.4$, $p_{3,4} = 155.5 \pm j\ 5802$
  $p_5 = 1479.1$

6) Elliptic high-pass filter derived from the low-pass filter:

- cutoff frequency = 1 kHz
- $M' = 65$, $N' = 5$, $K_s = 1$
- $m_{1,2} = 0 \pm j\ 4240$, $m_{3,4} = 0 \pm j\ 5750$
  $m_5 = 0$,
- $p_{1,2} = 182.3 \pm j\ 6799$, $p_{3,4} = 1555 \pm j\ 8763$
  $p_5 = 26689$

7) Elliptic band-pass filter derived from the low-pass filter:

- lower cutoff frequency = 700 Hz
- upper cutoff frequency = 1000 Hz
- $M' = 9$, $N' = 10$, $K_s = 1195.2$
- $m_{1,2} = 0 \pm j\ 6836$, $m_{3,4} = 0 \pm j\ 4043$
  $m_{5,6} = 0 \pm j\ 6386$, $m_{7,8} = 0 \pm j\ 4327$
  $m_9 = 0$,
  $p_{1,2} = 27{,}2 \pm j\ 6199$, $p_{3,4} = 222 \pm j\ 5252$
  $p_{5,6} = 130 \pm j\ 5951$, $p_{7,8} = 102 \pm j\ 4641$
  $p_{9,10} = 10.2 \pm j\ 4458$

8) Elliptic band-reject filter derived from the low-pass filter:

- lower cutoff frequency = 700 Hz
- upper cutoff frequency = 1000 Hz
- $M' = 10$, $N' = 10$, $K_s = 1$
- $m_{1,2} = 0 \pm j\ 5257$, $m_{3,4} = 0 \pm j\ 5931$
  $m_{5,6} = 0 \pm j\ 4659$, $m_{7,8} = 0 \pm j\ 6190$
  $m_{9,10} = 0 \pm j\ 4464$
- $p_{1,2} = 4003 \pm j\ 3407$
  $p_{3,4} = 22 \pm j\ 4335$, $p_{5,6} = 32.6 \pm j\ 6375$
  $p_{7,8} = 176.6 \pm j\ 4100$, $p_{9,10} = 290 \pm j\ 6729$

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"Méthode d'invariance à l'accélération pour dériver des filtres numériques récursifs" par A. Morin et P. Labbé

Ce document décrit une méthode pour concevoir des filtres numériques récursifs à partir des filtres analogiques lorsque le rapport entre le taux d'échantillonnage et la fréquence du pôle est faible. La fonction de transfert en z des filtres numériques obtenus par cette méthode, appelée l'invariance à l'accélération, a été déduite de la décomposition en fraction partielle de la fonction de transfert des filtres analogiques. Une fonction qui permet de calculer l'erreur entre la fonction de transfert numérique et la fonction de transfert analogique a aussi été déduite. Finalement, la performance de cette méthode est démontrée en traçant la réponse en fréquence des filtres Butterworth et elliptiques à ordres élevés.

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